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Motchenkova, E.I.

### ***published in***

Applied Economics Letters  
2014

### ***DOI (link to publisher)***

[10.1080/13504851.2013.861579](https://doi.org/10.1080/13504851.2013.861579)

### ***document version***

Publisher's PDF, also known as Version of record

[Link to publication in VU Research Portal](#)

### ***citation for published version (APA)***

Motchenkova, E. I. (2014). Cost minimizing sequential punishment policies for repeat offenders. *Applied Economics Letters*, 21(5), 360-365. <https://doi.org/10.1080/13504851.2013.861579>

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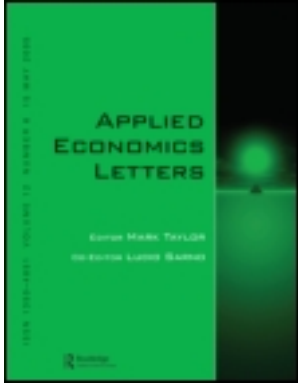
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## Applied Economics Letters

Publication details, including instructions for authors and subscription information:

<http://www.tandfonline.com/loi/rael20>

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Published online: 07 Jan 2014.

To cite this article: Evgenia Motchenkova (2014) Cost minimizing sequential punishment policies for repeat offenders, Applied Economics Letters, 21:5, 360-365, DOI: [10.1080/13504851.2013.861579](https://doi.org/10.1080/13504851.2013.861579)

To link to this article: <http://dx.doi.org/10.1080/13504851.2013.861579>

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# Cost minimizing sequential punishment policies for repeat offenders

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This article discusses optimal sanctions for repeat offenders. We analysed a multi-period decision problem, where the regulator's main objectives are to block any violations of law and to minimize the costs of crime control. We conclude that, when offenders are identical and wealth constrained, the government is resource constrained, can perfectly observe illicit gains and commits to a certain policy throughout the whole planning horizon, forward-looking solution implies that cost minimizing deterrence is decreasing in the number of offenses. This analysis is relevant in case when imprisonment is not commonly used, only monetary sanctions are allowed and limited liability of offenders plays an important role. The examples are tax evasion, violations of environmental regulations and violations of competition law.

**Keywords:** crime and punishment; crime prevention; repeat offenders

**JEL Classification:** K14; K42; C61

## 1. Introduction

The principle of escalating sanctions based on offense history is widely accepted and embedded in many penal codes and sentencing guidelines. Penalty escalation characterizes not only traditional crimes such as theft and murder, but also violations of competition law, violations of environmental regulations and tax evasion.

A number of recent contributions have tried to verify whether or not a sanction scheme that minimizes the (expected) social costs indeed has the property of sanctions increasing with offense history. So far it has been established in the law and economics literature that under certain circumstances escalating penalty schemes may be optimal. There are a number of studies, for example by Rubinstein (1979, 1980), Polinsky and Rubinfeld (1991), Polinsky and Shavell (1998), Baik and Kim (2001), Chu

*et al.* (2000), Miceli and Bucci (2005) or Endres and Rundshagen (2012), which provided justifications for escalating penalties. Other studies, however, show that for various reasons, optimal deterrence may actually involve declining penalties for repeat offenders (see, for example, Burnovski and Safra (1994); Dana (2001); Emons (2003) or Emons (2007)<sup>1</sup>).

Our analysis supports the second stream of literature. In this article we study the problem of optimal sanctions for repeat offenders in a multi-periods model that bridges  $n$ -period model of Burnovski and Safra (1994) and a two-period model suggested in Emons (2003). We extend the two-period result of declining penalties obtained in Emons (2003). While doing that we also generalize Burnovski and Safra (1994) study by deriving the optimal policy which is not only aimed at reducing the number of crimes, but also minimizes enforcement costs. In addition,

<sup>1</sup> Although, Emons (2007) finds partial support for both hypotheses.

our framework also allows to derive unconditional<sup>2</sup> result of declining penalties for repeat offenders, which was not achieved in Burnovski and Safra (1994).

## II. The Model

In this section we analyse an  $n$ -period repeated game, where the regulator's main objective is to block any violations of law and, at the same time, to minimize the costs of crime control. We describe a forward-looking solution, i. e. the offender and the regulator can commit to a certain strategy from the beginning of the game and the regulator does not change the parameters of the penalty scheme (fine and probability of control) till the end of the planning horizon. In that case we can consider a multi-period optimization problem of a cost minimizing regulator whose aim is to block violations of law. The solution of this problem gives the desired result of complete deterrence. Even the first crime never happens, unless benefits from crime are much higher than the initial wealth of the offender.

We consider a continuum of potential offenders which has measure 1. Individuals or firms live for  $n$  periods. In each period, the agents can engage in an illegal activity, such as polluting the environment or evading taxes. If an agent commits the act in either period he receives a monetary benefit  $b > 0$ . Following Polinsky and Rubinfeld (1991)  $b$  is the illicit gain and the crime creates no acceptable gain. The act causes a monetary harm  $h > 0$  to the society and, thus, has to be deterred. We assume that the following inequality is satisfied,  $h > b$ , so that the act is not socially desirable.

To achieve deterrence the government chooses sanctions and a probability of apprehension. The regulator cannot tell in which period of its life the individual is. It can only observe the information after the crime has been discovered. Hence, the regulator only observes whether the crime is the first or second or  $n^{\text{th}}$  one. Accordingly, the government applies fines  $s_1, s_2, \dots, s_n \geq 0$ , where  $s_i$  is the penalty in case the offense by this particular agent is recorded by the authority already  $i$  times. Moreover, the government chooses a rate of law enforcement,  $p$ , which can also be seen as the probability of apprehension. We assume that  $p$  is the same for all (first time and repeated) offenses. Since apprehension is costly, the government wishes to minimize  $p$ .

The overall objective of the regulator is to reduce the number of crimes. Subject to that objective being reached, the regulator aims to minimize costs of control,  $p$ . So, the

objective function of the regulator can be written as  $\max - (p + Hk)$ , where  $p$  is the probability of control (or rate of law enforcement),  $k$  is the number of crimes, and  $H$  is the disutility from crime for the regulator, which is assumed to be a large positive number.

The agents are risk neutral and maximize expected income. They have initial wealth  $W > 0$  and hold it over all  $n$  periods unless the government interferes with sanctions. We may think of  $W$  as the value of the privately owned house or assets with a long maturity. Any additional income that agents receive in any of the periods, be it through legal or illegal activity, is consumed immediately, and the maximum of what the government can extract from the agents is  $W$ .<sup>3</sup> If the fine exceeds the agent's wealth, he goes bankrupt and the government seizes the remaining assets. This implies that the fines  $s_1, s_2, \dots, s_n$  have to satisfy the 'budget constraint'  $\sum_{i=1}^n s_i = W$ . To simplify the analysis we also assume no discounting.

An agent chooses the number of crimes that can be committed or, in other words, he can choose between the following strategies:

Not to commit a criminal act at all. Then, the utility from this strategy for the 'offender' has the following form  $U(0, 0, \dots, 0) = W$ .

Commit crime only once in any of the periods. The utility for the offender equals  $U(1, 0, \dots, 0) = W + b - ps_1$ .

Commit crime in any two periods:  $U(1, 1, 0, \dots, 0) = W + b - ps_1 + b - p(1-p)s_1 - p^2s_2$ .

Commit crime in any three periods:  $U(1, 1, 1, \dots, 0) = U(0, 1, 1, 1, \dots, 0) = U(0, 0, \dots, 0, 1, 1, 1) = W + b - ps_1 + b - p(1-p)s_1 - p^2s_2 + b - (1-p)^2ps_1 - 2p^2(1-p)s_2 - p^3s_3$ .

Commit crime in all  $n$  periods:  $U(1, 1, 1, \dots, 1) = W + b - ps_1 + b - p(1-p)s_1 - p^2s_2 + b - (1-p)^2ps_1 - 2p^2(1-p)s_2 - p^3s_3 + \dots + b - (C_{n-1}^0(1-p)^{n-1}ps_1 + C_{n-1}^1(1-p)^{n-2}p^2s_2 + C_{n-1}^2(1-p)^{n-3}p^3s_3 + \dots + C_{n-1}^{n-2}(1-p)p^{n-1}s_{n-1} + C_{n-1}^{n-1}p^ns_n)$ ,

where coefficients of these polynomials are formed according to the following formula:

$$C_h^k = \frac{h!}{k!(h-k)!}, \quad h \geq k$$

We impose the following assumptions on the parameters  $0 < p < 1, b > 0, W > 0$ . The possibility  $p = 0$  does not make sense, since then there is no threat for the agent to be convicted and no way to prove the criminal to be guilty. We also assume that agents have enough wealth so that

<sup>2</sup> Without restriction on probability of detection being sufficiently small.

<sup>3</sup> This assumption may seem to be restrictive. In most of the cases, for example, in the case of tax evasion or illegal price-fixing activities, the penalty takes into account not only initial wealth of the firm but also accumulated rents from illegal activities. However, this assumption is adopted here in order to focus on obtaining analytical results with respect to establishing an optimal sequence of sanctions.

deterrence is always possible, i.e.  $nb < \sum_{i=1}^n s_i \leq W$ .

Further, we derive sanctions that give the agents the proper incentives not to engage in criminal activities in either period. This means, we derive a penalty scheme which ensures  $U(1, 0, \dots, 0) < U(0, 0, \dots, 0)$ ,  $U(1, 1, \dots, 0) < U(0, 0, \dots, 0)$ , ...,  $U(1, 1, \dots, 1) < U(0, 0, \dots, 0)$ . These are included as constraints in the following optimization model. The aim of the regulator to prevent crime and to minimize the enforcement costs is reflected in the objective function (1) below, whereas the aim to provide incentives for the agents not to commit any crime is reflected in incentive constraints (2)–(n + 1).

$$\min p + Hk \quad (1)$$

s.t.

$$b - ps_1 \leq 0 \quad (2)$$

$$2b - ps_1 - p(1-p)s_1 - p^2s_2 \leq 0 \quad (3)$$

$$3b - ps_1 - p(1-p)s_1 - p^2s_2 - (1-p)^2ps_1 - 2p^2(1-p)s_2 - p^3s_3 \leq 0 \quad (4)$$

$$lb - \sum_{h=1}^l \sum_{k=1}^h C_{h-1}^{k-1} (1-p)^{h-k} p^k s_k \leq 0 \quad (l+1)$$

$$nb - \sum_{h=1}^n \sum_{k=1}^h C_{h-1}^{k-1} (1-p)^{h-k} p^k s_k \leq 0 \quad (n+1)$$

$$s_1 + s_2 + \dots + s_{n-1} \leq W \quad (n+2)$$

$$s_1 \geq 0, s_2 \geq 0, \dots, s_{n-1} \geq 0, p > 0 \quad (n+3)$$

The Lagrangian for this problem has the following form:

$$L = -p - \sum_{j=1}^n \lambda_j [jb - \sum_{h=1}^j \sum_{k=1}^h C_{h-1}^{k-1} (1-p)^{h-k} p^k s_k] - \lambda^* (s_1 + s_2 + \dots + s_{n-1} - w) \quad (5)$$

Using Kuhn–Tucker conditions to solve the minimization problem (1)–(n + 3), we obtain the result stated in Proposition 1.

**Proposition 1:** *The optimal cost minimizing sanction scheme sets the penalty for the first detected violation equal to the entire wealth of the agent, and for all subsequent violations the penalties will be equal to zero, i.e.  $s_1^* = W$  and  $s_2^* = \dots = s_n^* = 0$ . The probability of law enforcement is constant over time and equals  $p^*$ , which represents the smallest positive solution of the polynomial of order  $n$  in  $p$ , given by expression (A10).*

The proof of Proposition 1 consists of several steps: first, we derive first-order conditions (FOCs) and complementary slackness conditions of the minimization problem described above; second, based on the FOCs we prove Lemma 3, which states that inequality  $\frac{\partial L}{\partial s_l} > \frac{\partial L}{\partial s_{l+1}}$  holds for any time period  $l \in \{1, \dots, n-1\}$ ; finally, applying Lemma 3 and the complementary slackness conditions we obtain the optimal penalty schedule with  $s_1^* = W$  and  $s_2^* = \dots = s_n^* = 0$  and  $p > 0$ .

For detailed proof see Appendix.

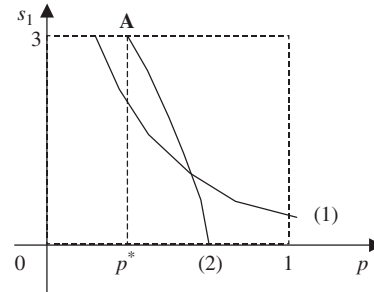
The intuition behind this proposition follows from the incentive constraints. The agent pays the sanction  $s_1$  with probability  $p$ , while any further sanction will be paid with lower probability. Hence, since paying the first fine is more likely than paying any subsequent fine, shifting resources from the last periods to  $s_1$  increases deterrence for given  $p$ . Consequently, as in Emons (2003),  $p$  is minimized by putting all scarce resources into  $s_1$ .

**Example 2:** Figure 1 illustrates the proof graphically in the  $(p, s_1)$  diagram for the two-period case. The game in this case is described as follows. A strategy of player 1 (regulator) is given by  $\sigma = (p, s_1, s_2)$ , whereas the strategy set of player 2 (offender, or firm) is given by  $\{0, 1, 2\}$ .

In case  $n = 2$ , the optimization problem of the regulator will be as follows:

$$\min p + Hk$$

$$\text{s.t. } b - ps_1 \leq 0 \quad (1), \quad 2b - ps_1 - p(1-p)s_1 - p^2s_2 \leq 0 \quad (2), \quad s_1 + s_2 \leq W \quad (3), \quad 0 < p \leq 1$$



**Fig. 1.** Graphical illustration of solution in two-period case. Parameter values  $b = 1$ ,  $W = 3$

Taking into account that  $b > 0$ ,  $W > 2b$ ,  $s_2 = W - s_1$ , the solution of this problem, which has the form  $s_1^* = W$ ,  $s_2^* = 0$ ,  $p = p^* > 0$ , is represented by point A in Fig. 1.

### III. Conclusions

We find that, when offenders are identical and wealth constrained, illicit gains are perfectly observable, the government is resource constrained and can commit to a certain policy throughout the whole planning horizon, forward-looking solution implies that cost minimizing deterrence is decreasing, rather than increasing, in the number of offenses. We prove that for the agents who may commit an act several times, optimal sanctions are such that the fine for the first crime equals the offender's entire wealth, and the fines are zero for all the subsequent crimes. Since the agent can only be a repeat offender if he has been a first-time offender, there are no further offenses if we completely deter the first one. This conclusion also supports the result obtained by Emons (2003) for a two-period model.

This result contradicts the widely prevailing escalating penalties imbedded in many penal codes and sentencing guidelines and might be of limited applicability due to restrictive assumptions we make. However, the analysis presented in this article is particularly relevant for a number of violations for which imprisonment is not commonly used, only monetary sanctions are allowed, and limited liability of offenders plays an important role. This is the case for, for example, tax evasion, violations of environmental regulations and violations of competition law in Europe.<sup>4</sup> In those settings, assumption that agents are wealth constrained becomes particularly important.

### Acknowledgements

The author thanks Eric van Damme, Peter M. Kort, Dolf Talman, Winand Emons, Quan Wen, Jacco Wielhouwer and anonymous referees for stimulating discussions and valuable comments.

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<sup>4</sup> This may also be applicable for other 'economic crimes' and especially in the corporate environment. For extensive discussion of those examples see Burnovski and Safra (1994).

## Appendix

### Proof of Proposition 1:

**Proof.** To derive the FOCs we take partial derivatives of expression (5) with respect to all  $n - 1$  variables, which denote the penalties in the corresponding periods. Recall that, taking into account that the budget constraint must be binding,  $s_n$  can be expressed through all the unknowns and initial wealth as follows  $s_n = W - \sum_{i=1}^{n-1} s_i$ .

So, differentiating and simplifying the expressions, we obtain  $n - 1$  FOCs w.r.t. penalties (A1)–(A4) and one FOC w.r.t. probability of law enforcement (A9).  $n + 1$  complementary slackness conditions are reflected in expressions (A5)–(A8) below.

$$\begin{aligned} \frac{\partial L}{\partial s_1} &= p(1-p)^0 \sum_{i=k+1}^n \lambda_i \\ &+ \sum_{k=1}^{n-1} \left[ C_k^0 p(1-p)^k \left( \sum_{i=k+1}^n \lambda_i \right) \right] \\ &- \lambda_n p^n - \lambda^* \leq 0 \quad (= 0 \text{ if } s_1 > 0) \end{aligned} \quad (\text{A1})$$

$$\begin{aligned} \frac{\partial L}{\partial s_2} &= \sum_{k=1}^{n-1} \left[ C_k^1 p^2 (1-p)^{k-1} \left( \sum_{i=k+1}^n \lambda_i \right) \right] \\ &- \lambda_n p^n - \lambda^* \leq 0 \quad (= 0 \text{ if } s_2 > 0) \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} \frac{\partial L}{\partial s_l} &= \sum_{k=l-1}^{n-1} \left[ C_k^{l-1} p^l (1-p)^{k-(l-1)} \left( \sum_{i=k+1}^n \lambda_i \right) \right] \\ &- \lambda_n p^n - \lambda^* \leq 0 \quad (= 0 \text{ if } s_l > 0) \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} \frac{\partial L}{\partial s_{n-1}} &= \sum_{k=n-2}^{n-1} \left[ C_k^{n-2} p^{n-1} (1-p)^{k-(n-2)} \left( \sum_{i=k+1}^n \lambda_i \right) \right] \\ &- \lambda_n p^n - \lambda^* \leq 0 \quad (= 0 \text{ if } s_{n-1} > 0) \end{aligned} \quad (\text{A4})$$

Complementary slackness conditions are

$$\lambda_1 \geq 0 \quad (\lambda_1 * (2) = 0) \quad (\text{A5})$$

$$\lambda_2 \geq 0 \quad (\lambda_2 * (3) = 0) \quad (\text{A6})$$

$$\lambda_n \geq 0 \quad (\lambda_n * (n+1) = 0) \quad (\text{A7})$$

$$\lambda^* \geq 0 \quad \left( \lambda^* * \left( \sum_{i=1}^{n-1} s_i - W \right) = 0 \right) \quad (\text{A8})$$

$$\frac{\partial L}{\partial p} = 0 \quad (\text{A9})$$

Next, we prove the following lemma.

**Lemma 3:** For any  $l \in \{1, \dots, n-1\}$  and  $p < \frac{1}{2}$ , we have that  $\frac{\partial L}{\partial s_l} < \frac{\partial L}{\partial s_{l+1}}$ .

**Proof.** The proof of this lemma is based on mathematical induction argument and is available from the author upon request.

Next, using the result of Lemma 3, we derive the optimal penalty schedule.

We start by showing that constraint  $(n+2)$  is always binding.

In case this constraint is not binding, there are three possibilities:

- (1)  $\sum_{i=1}^{n-1} s_i < W$  and  $s_i > 0$  for all  $i \in \{1, \dots, n-1\}$ ,
- (2)  $\sum_{i=1}^{n-1} s_i < W$  and  $s_i = 0$  for all  $i \in \{1, \dots, n-1\}$ ,
- (3)  $\sum_{i=1}^{n-1} s_i < W$  and  $s_i = 0$  for some  $i \in \{1, \dots, n-1\}$ .

The result of Lemma 3 immediately implies that the solution with  $s_i > 0$  for all  $i \in \{1, \dots, n-1\}$  is impossible.

Consider  $\sum_{i=1}^{n-1} s_i < W$  and  $s_i = 0$  for all  $i \in \{1, \dots, n-1\}$ . Then, the first-order conditions (A1)–(A4) imply that  $\frac{\partial L}{\partial s_i} < 0$  for all  $i \in \{1, \dots, n-1\}$ . Moreover,  $\lambda^* = 0$ . However, then, considering the last period  $n-1$ , we obtain that  $\frac{\partial L}{\partial s_{n-1}} = \sum_{k=n-2}^{n-1} [C_k^{n-2} p^{n-1} (1-p)^{k-(n-2)} (\sum_{i=k+1}^n \lambda_i)] - \lambda_n p^n = p^{n-1} \lambda_{n-1} + n p^{n-1} \lambda_n (1-p) > 0$ . Hence, condition (A4) cannot be strictly negative. This implies that the outcome with  $s_i = 0$  for all  $i \in [1, n-1]$  and  $\lambda^* = 0$  also cannot arise as a solution of the minimization problem of the regulator.

Next, consider  $\sum_{i=1}^{n-1} s_i < W$  and  $s_i = 0$  for some  $i \in \{1, \dots, n-1\}$ . Assume  $s_l = 0$  for  $l < n-1$ . This means that Equation A3 must be nonpositive. But we have just shown that  $\frac{\partial L}{\partial s_{n-1}} > 0$ . Hence, using Lemma 3, we can conclude that this outcome also cannot be a solution.

The outcome with  $\sum_{i=1}^{n-1} s_i = W$  and  $s_i = 0$  for  $i < k \in \{1, \dots, n-1\}$  and  $s_l > 0$  for  $l < k \in \{1, \dots, n-1\}$  is impossible due to the result of Lemma 3.



Moreover, the outcome with  $\sum_{i=1}^{n-1} s_i = W$  and  $s_1 > 0$ ,  $s_2 > 0$  and  $s_i = 0$  for all  $i \in \{3, \dots, n-1\}$  cannot arise. Consider  $s_1 > 0$ ,  $s_2 > 0$ . Using Equations A1 and A2 we obtain that  $\frac{\partial L}{\partial s_1} = \frac{\partial L}{\partial s_2} = 0$ . But, this again contradicts the result of Lemma 3.

We conclude that only the following is possible:  $s_1^* > 0, s_2^* = \dots = s_n^* = 0$  and  $\sum_{i=1}^{n-1} s_i = W$ , which implies that  $s_1^* = W, s_2^* = \dots = s_n^* = 0$ .

Finally, optimal behaviour implies that only condition  $(n+1)$  on the benefits from crime will be binding, so that

$\lambda_1 = \lambda_2 = \dots = \lambda_{n-1} = 0$  and  $\lambda_n \geq 0$ .<sup>5</sup> Hence, the expressions for the optimal probability of law enforcement,  $p^*, \lambda^*$  and  $\lambda_n$  will be determined from conditions  $(n+1)$ ,  $\frac{\partial L}{\partial s_1} = 0$  and  $\frac{\partial L}{\partial p} = 0$ . In particular,  $p^*$  is given by the solution of the polynomial of order  $n$  specified in expression (A10) below with  $s_1 = W, s_2 = 0, \dots, s_n = 0$ .

$$nb - \sum_{h=1}^n \sum_{k=1}^h C_{h-1}^{k-1} (1-p)^{h-k} p^k s_k = 0 \quad (\text{A10})$$

End of the proof of Proposition 1.

<sup>5</sup> The fact that only constraint  $(n+1)$  on the benefits from crime is binding when  $s_1^* = W, s_2^* = \dots = s_n^* = 0$  can be proven by contradiction. Detailed proof is available from the author upon request. The intuition is as follows. Assume, for example, that constraint  $(l+1)$  is binding for some  $l \in \{2, \dots, n-1\}$ , then it follows that the LHS of the constraint  $(l+2)$  has to be strictly positive, which is impossible by construction of the problem.